

Thermoacoustic model of a modified free piston Stirling engine with a thermal buffer tube

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ABSTRACT

This article presents a modified free-piston Stirling heat engine configuration in which a thermal buffer tube is added to sandwich between the hot and cold heat exchangers. Such a modified configuration may lead to an easier fabrication and lighter weight of a free piston. To analyze the thermodynamic performance of the modified free piston Stirling heat engine, thermoacoustic theory is used. In the thermoacoustic modelling, the regenerator, the free piston, and the thermal buffer tube are given at first. Then, based on linear thermoacoustic network theory, the thermal and thermodynamic networks are presented to characterize acoustic pressure and volume flow rate distributions at different interfaces, and the global performance such as the power output, the heat input and the thermal efficiency. A free piston Stirling heat engine with several hundreds of watts mechanical power output is selected as an example. The typical operating and structure parameters are as follows: frequency around 50 Hz, mean pressure around 3.0 MPa, and a diameter of free piston around 50 mm. From the analysis, it was found that the modified free-piston Stirling heat engine has almost the same thermodynamic performance as the original design, which indicates that the modified configuration is worthy to develop in future because of its mechanical simplicity and reliability.

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1. Introduction

The Stirling engine is driven by the cyclic hot expansion and cold contraction of the working gas. It is one of the most efficient heat engines. The Stirling engine is an external combustion engine, and many kinds of heat sources could be used such as solar energy, industrial waste heat and biomass energy. In addition, the Stirling engines have advantages of smooth operation, low noise, and environmental friendliness [1].

A free piston Stirling engine (FPSE) is a variable form of Stirling machines by eliminating the crank-shaft device of the traditional Stirling engine. Its thermodynamic performance is greatly influenced by the moving mass and spring stiffness of the displacer assembly (i.e. free piston) [2]. Compared with the traditional Stirling engine, it could be self-started and has better reliability, higher efficiency as well as lower vibration. Therefore, the free piston Stirling engine is getting more and more attentions and might have many applications [3–6].

However, there are some drawbacks for the current FPSE. A large part of the displacer works in the high temperature region, which leads to several disadvantages including difficult manufac-

turing, assembling and sealing, etc. In order to overcome these disadvantages, a modified FPSE with a thermal buffer tube is proposed. In this new configuration, the high-temperature part of the displacer is replaced by a thermal buffer tube. Meanwhile, several layers of anti-radiation stainless steel meshes are stacked in the thermal buffer tube to reduce the radiation heat loss. In addition, an ambient heat exchanger is set at the ambient region of the displacer to make sure that the displacer operate at ambient temperature. This configuration can effectively reduce the difficulty of manufacturing, assembling and sealing of the displacer. Also, the lifetime of the displacer would be greatly improved.

2. Modified free piston Stirling engine

Fig. 1 shows the schematic of a conventional FPSE. The main components of the FPSE include: a hot end heat exchanger, a regenerator, an ambient heat exchanger, a displacer, a power piston, and a linear alternator. As the engine works, part of the acoustic power from the expansion space is used to drive the mechanical device. The other acoustic power enters the ambient end of the regenerator, and then amplifies.

Fig. 2 shows the schematic of a modified FPSE. It uses a thermal buffer tube to replace the high-temperature part of the displacer. Its working principle is the same as a conventional engine. The

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Nomenclature

English letters

i	$\sqrt{-1}$
X	displacement amplitude (m)
M	moving mass (kg)
r_h	hydraulic radius (m)
S	cross area of regenerator (m^2)
T_h	heating temperature (K)
V	volume of thermo buffer tube (m^3)
d	diameter of thermo buffer tube (m)
k	spring stiffness (N/m)
l	length of regenerator (m)
$\bar{p}$	amplitude of pressure wave (MPa)
R_m	damping coefficient (kg/s)
T_0	ambient temperature (K)
$\bar{U}$	amplitude of volume flow rate wave (m^3/s)
W	acoustic power (W)

Greek letters

τ	temperature ratio
μ_0	dynamic viscosity (kg/m s)
θ	phase ($^\circ$)
γ	adiabatic index
η	thermo efficiency
ϕ	porosity
ω	angular frequency

Superscript

*	conjugate
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Subscript

$i(1-4)$	status points
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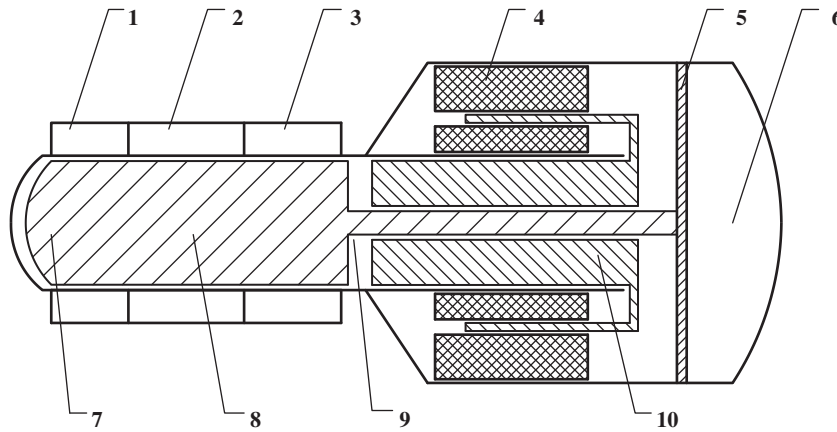


Fig. 1. Schematic of conventional FPSE (1. hot end heat exchanger, 2. regenerator, 3. ambient heat exchanger, 4. linear alternator, 5. planar spring, 6. back space, 7. expansion space, 8. displacer, 9. compression space, 10. power piston).

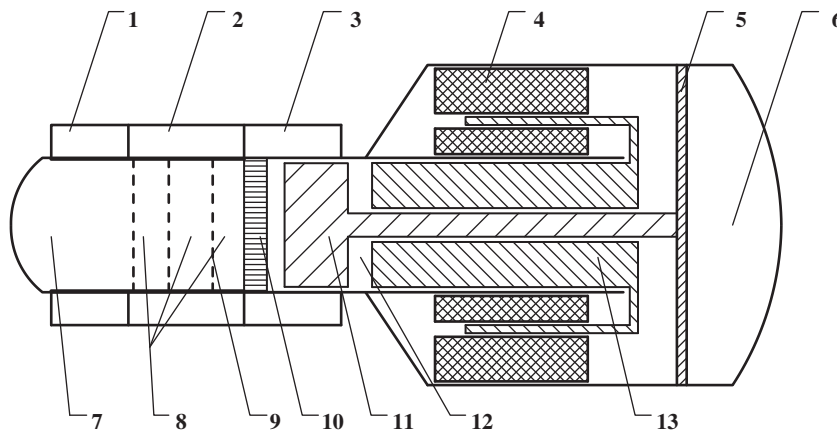


Fig. 2. Schematic of modified FPSE (1. hot heat exchanger, 2. regenerator, 3. ambient heat exchanger, 4. linear alternator, 5. planar spring, 6. back space, 7. expansion space, 8. thermal buffer tube, 9. mesh, 10. ambient releaser, 11. displacer, 12. compression space, 13. power piston).

performance of the displacer is determined by the diameter of the rod, the spring stiffness of the planar flexure and the damping coefficient. The thermal buffer tube is a circular tube. The working gas in the thermal buffer tube acts as a long gas piston, transporting the acoustic power in forms of pressure and velocity waves from the hot region to the ambient region.

However, there are several problems emerging with the introduction of the thermal buffer tube. First, the radiation heat loss from the hot region to the ambient region will increase significantly. Worse more, the gas convection occurring in the thermal buffer tube would deteriorate the heat loss therein. To reduce the heat loss, several layers of stainless steel meshes are placed

in the thermal buffer to decrease the radiation heat loss and make the flow uniform. Meanwhile, an ambient releaser is introduced to make sure that the displacer work in the ambient temperature.

3. Mathematical model

For all kind of oscillating flow heat engines, the dynamic feature plays an important role. In the following analysis, assuming that the working gas is helium and the heat transfer process is ideal. Then, the dynamic models of the regenerator, the thermal buffer tube, and the displacer are established, respectively. Since that attention is focused on how the heat is converted into acoustic power efficiently in the thermodynamic loop of the engine, the coupling between the engine and the linear alternator is not considered in this work.

Fig. 3 shows a simplified flow chart of the modified FPSE with a thermal buffer tube. The pressure wave and the volume flow rate wave at the inlet of the regenerator are $(\tilde{p}_1, \tilde{U}_1)$, the parameters of the points 2–5 have the same symbols as point 1 except for the subscripts.

It is known that from the momentum and mass balances:

$$\begin{cases} \tilde{p}_1 = \tilde{p}_4 = \tilde{p}_5 \\ \tilde{U}_1 = \tilde{U}_4 - \tilde{U}_5 \end{cases} \quad (1)$$

The dynamic model of the regenerator [7] is:

$$\begin{pmatrix} \tilde{p}_2 \\ \tilde{U}_2 \end{pmatrix} = \begin{pmatrix} FPP1 & FPU1 \\ FUP1 & FUU1 \end{pmatrix} \begin{pmatrix} \tilde{p}_1 \\ \tilde{U}_1 \end{pmatrix} \quad (2)$$

where

$$FPP1 = 1 + \frac{i\omega C_0 R_0}{2} g(\tau, b), \quad FPU1 = -\frac{R_0(\tau + 1)}{2} f(\tau, b)$$

$$FUP1 = -i \frac{\omega C_0 \tau}{\tau - 1} \ln \tau, \quad FUU1 = \tau$$

$$\tau = \frac{T_h}{T_0}, \quad b = 0.68, \quad R_0 = \frac{6\mu_0 l}{S \cdot r_h^2}, \quad C_0 = \frac{\phi S l}{p_0}$$

$$f(\tau, b) = \frac{2}{b+2} \left[\frac{\tau^{b+2} - 1}{\tau^2 - 1} \right],$$

$$g(\tau, b) = \frac{2}{b+2} \left[\frac{\tau^{b+2} \ln \tau - (\tau^{b+2} - 1)/(b+2)}{(\tau - 1)^2} \right]$$

The dynamic model of the thermal buffer tube [8] is:

$$\begin{pmatrix} \tilde{p}_3 \\ \tilde{U}_3 \end{pmatrix} = \begin{pmatrix} FPP2 & FPU2 \\ FUP2 & FUU2 \end{pmatrix} \begin{pmatrix} \tilde{p}_2 \\ \tilde{U}_2 \end{pmatrix} \quad (3)$$

where

$$FPP2 = 1, \quad FPU2 = 0$$

$$FUP2 = -i \frac{\omega V}{\gamma p_0}, \quad FUU2 = 1$$

The dynamic model of the displacer is:

$$\begin{pmatrix} \tilde{p}_4 \\ \tilde{U}_4 \end{pmatrix} = \begin{pmatrix} FPP3 & FPU3 \\ FUP3 & FUU3 \end{pmatrix} \begin{pmatrix} \tilde{p}_3 \\ \tilde{U}_3 \end{pmatrix} \quad (4)$$

where

$$FPP3 = \frac{A_3}{A_4}, \quad FPU3 = -\frac{1}{A_3 A_4} \left[R_m + i\omega \left(M - \frac{k}{\omega^2} \right) \right]$$

$$FUP3 = 0, \quad FUU3 = \frac{A_4}{A_3}$$

A_3 and A_4 represent the area of the inlet and outlet of the displacer, respectively.

From Eqs. (2) and (3):

$$\begin{pmatrix} \tilde{p}_4 \\ \tilde{U}_4 \end{pmatrix} = \begin{pmatrix} FPP & FPU \\ FUP & FUU \end{pmatrix} \begin{pmatrix} \tilde{p}_1 \\ \tilde{U}_1 \end{pmatrix} \quad (5)$$

From Eqs. (1)–(5):

$$\begin{cases} \tilde{U}_1 = \frac{1-FPP}{FPU} \tilde{p}_1 \\ \tilde{U}_2 = FUP1 * \tilde{p}_1 + FUU1 * \tilde{U}_1 \\ \tilde{p}_2 = FPP1 * \tilde{p}_1 + FPU1 * \tilde{U}_1 \\ \tilde{U}_3 = FUP2 * \tilde{p}_2 + FUU2 * \tilde{U}_2 \\ \tilde{p}_3 = FPP2 * \tilde{p}_2 + FPU2 * \tilde{U}_2 \\ \tilde{U}_4 = FUP * \tilde{p}_1 + FUU * \tilde{U}_1 \\ \tilde{U}_5 = \tilde{U}_4 - \tilde{U}_1 \\ \tilde{p}_5 = \tilde{p}_4 = \tilde{p}_1 \end{cases} \quad (6)$$

From Eq. (6), it indicates that once the pressure wave of the inlet of the regenerator is known, the remaining parameters could be calculated easily. Then, the acoustic power and the thermal efficiency could be calculated as follows:

$$W_2 = \frac{1}{2} \text{Re}(\tilde{p}_2 \cdot \tilde{U}_2^*), \quad W_5 = \frac{1}{2} \text{Re}(\tilde{p}_5 \cdot \tilde{U}_5^*), \quad \eta = \frac{W_5}{W_2} \quad (7)$$

4. Simulation results and discussion

A modified free piston Stirling engine with hundreds of watts acoustic power output is proposed and studied in this article. The effect of the size of the thermal buffer tube on the engine

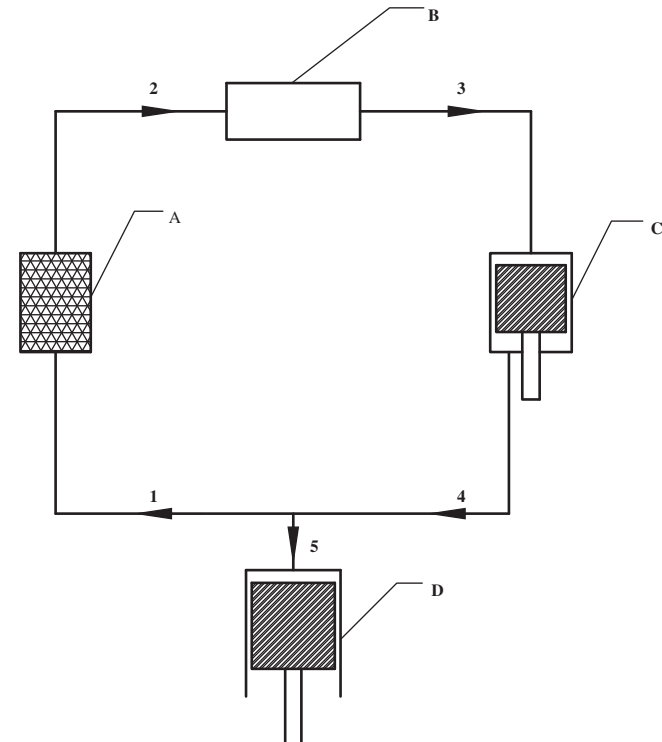


Fig. 3. Flow chart of FPSE with a thermal buffer tube (A. regenerator, B. thermal buffer tube, C. displacer, D. power piston; 1. inlet of the regenerator, 2. outlet of the regenerator, 3. inlet of the displacer, 4. outlet of the displacer, 5. outlet of the engine).

performance is investigated. Table 1 shows its dimensions and operating parameters.

We set the length of the thermal buffer tube being 70 mm and the diameter of the thermal buffer tube varying from 0 to 50 mm. The following figures show the effect of the thermal buffer size on the thermodynamic performance.

Fig. 4 shows the curve of the output acoustic power versus the diameter of the thermal buffer tube. As seen in Fig. 4 that as the diameter increases, the output acoustic power decreases. The thermal efficiency and relative Carnot efficiency slightly decrease as well (Fig. 5). The output phase and volume flow rate increase apparently (Figs. 6 and 7). It can be seen that the output acoustic power drops slightly although the phase gets worse, which could be attributed to the increase of output volume flow rate. However, it is adverse to match a linear alternator, so the size of the thermal buffer tube should not be too large.

On the other hand, the displacement of the working gas in the thermal buffer tube should not be too large; otherwise, there will be a huge heat loss. Define X_2 and X_3 as the amplitude of the working gas displacement at the inlet and outlet of the thermal buffer tube respectively. Their expressions are

$$X_2 = \frac{4|U_2|}{\pi\omega d^2}, \quad X_3 = \frac{4|U_3|}{\pi\omega d^2} \quad (8)$$

Their values are listed in Table 2.

Referring to the experiences of a pulse tube refrigerator, the gas displacement usually does not exceed 10% of the length of the thermal buffer tube (pulse tube) to avoid large heat loss. Since the length of the thermal buffer tube is 70 mm, the displacement should be less than 7 mm. As indicated in Table 2, the displacement is too large when the diameter is less than 40 mm. From Figs. 4–7, it can also be found that the output power and the coupling with a linear alternator will be seriously affected under this condition. Therefore, to avoid large heat loss, the size of the thermal buffer tube should not be too small.

Table 1
Structure and operating parameters.

Structure parameters		Operating parameters	
Diameter of displacer	50 mm	Heating temperature	923.15 K
Diameter of rod	20 mm	Ambient temperature	303.15 K
Diameter of regenerator	33.5 mm	Working gas	helium
Length of regenerator	40 mm	Working pressure	3 MPa
Wire diameter	0.051 mm	Pressure wave of the inlet of regenerator	0.3 MPa
Mesh number	180	Frequency	50 Hz
Porosity	0.7	Damping coefficient	25 kg/s
Moving mass	0.185 kg	Spring stiffness	15 kN/m

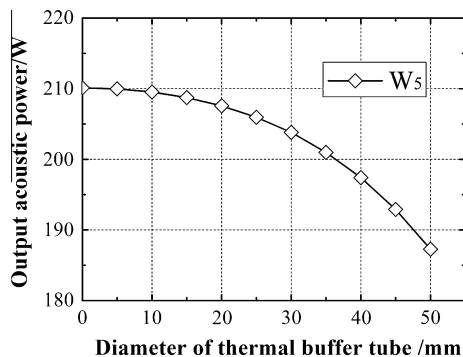


Fig. 4. Output acoustic power vs. diameter of the thermal buffer tube.

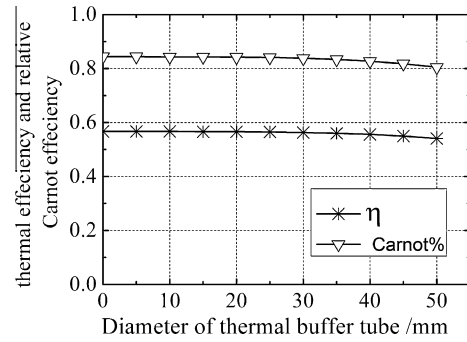


Fig. 5. Thermal efficiency vs. diameter of the thermal buffer tube.

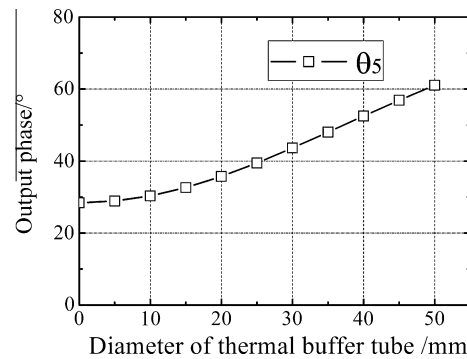


Fig. 6. Output phase vs. diameter of the thermal buffer tube.

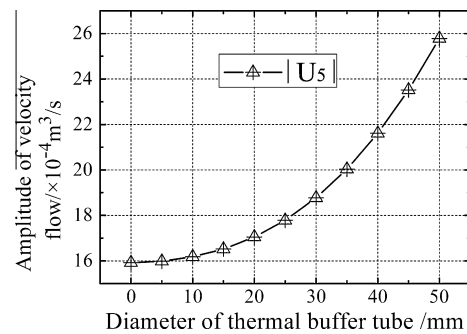


Fig. 7. Output volume flow rate vs. diameter of the thermal buffer tube.

Table 2
Gas displacement amplitude at inlet and outlet of the thermal buffer tube.

d (mm)	X_2 (mm)	X_3 (mm)
5	455.89	456.82
10	113.44	114.38
15	50.04	50.99
20	27.86	28.82
25	17.61	18.59
30	12.06	13.07
35	8.73	9.76
40	6.59	7.65
45	5.15	6.23
50	4.14	5.24

5. Conclusions

A modified FPSE with a thermal buffer tube is proposed in this article. Moreover, a simplified thermoacoustic model is established for the new design. Based on the model, the effects of the thermal buffer tube size on the performance of the modified FPSE are investigated in detail. The analysis indicates that the thermal buffer tube would not significantly decrease the acoustic power output but keep a relatively high efficient operation. Taking into the consideration of several advantages of the modified FPSE, this new configuration is worthy of testing and developing in future.

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